Prove that $\frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5n-4)\times(5n+1)}=\frac{n}{5n+1}$ for all integers $n\geq 1$ **SCORE: _____ / 10 PTS**

by mathematical induction, showing all steps demonstrated in lecture.

BASIS STEP:

If $n=1$, $\boxed{\frac{1}{1\times 6}=\frac{1}{6} \qquad \frac{1}{5(1)+1}=\frac{1}{6}}$ 1

INDUCTIVE STEP:

1 $\boxed{\text{Assume } \frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5k-4)\times(5k+1)}=\frac{k}{5k+1}}$ $\boxed{\text{for some arbitrary but particular integer } k}$ 1

2 $\boxed{\frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5k-4)\times(5k+1)}+\frac{1}{(5k+1)\times(5k+6)}}$

1 $\boxed{=\frac{k}{5k+1}+\frac{1}{(5k+1)(5k+6)}}$

FIRST & LAST 2 TERMS MUST BOTH BE SHOWN

$=\frac{k(5k+6)+1}{(5k+1)(5k+6)}$

1 $\boxed{=\frac{5k^2+6k+1}{(5k+1)(5k+6)}}$

1 $\boxed{=\frac{(5k+1)(k+1)}{(5k+1)(5k+6)}}$

$=\frac{k+1}{5k+6}$

1 $\boxed{=\frac{k+1}{5(k+1)+1}}$

1 $\boxed{\text{By mathematical induction, } \frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5n-4)\times(5n+1)}=\frac{n}{5n+1} \text{ for all integers } n\geq 1}$

Evaluate $\sum_{k=1}^{200}(8k-3k^2)$. **Your final answer must be a number (not involving arithmetic operations).** SCORE: _____ / 4 PTS

$$= \underbrace{8\sum_{k=1}^{200} k - 3\sum_{k=1}^{200} k^2}\ (1)$$

$$= 8\cdot \underbrace{\tfrac{1}{2}(200)(201)}\ (1) - 3\cdot \underbrace{\tfrac{1}{6}(200)(201)(401)}\ (1)$$

$$= \underline{-7899300}\ (1)$$

If $f(x)=x^4$, expand and **completely simplify** the difference quotient $\dfrac{f(x+h)-f(x)}{h}$. $2\frac{1}{2}$

SCORE: _____ / 4 PTS

$$\frac{(x+h)^4-x^4}{h} = \frac{\cancel{x^4}+4x^3h+6x^2h^2+4xh^3+h^4-\cancel{x^4}}{h}$$

$$= 4x^3+6x^2h+4xh^2+h^3 \quad 1\tfrac{1}{2}$$

Expand and **completely simplify** the complex number $(3-2i)^5$.

SCORE: _____ / 7 PTS

$$1(3)^5(-2i)^0 + \underbrace{5(3)^4(-2i)^1}_{\frac{1}{2}} + \underbrace{10(3)^3(-2i)^2}_{\frac{1}{2}} + \underbrace{10(3)^2(-2i)^3}_{\frac{1}{2}} + \underbrace{5(3)^1(-2i)^4}_{} \ \tfrac{1}{2} + 1(3)^0(-2i)^5$$

$$= \underbrace{243}_{\frac{1}{2}} + 5(81)(-2i) + 10(27)(-4) + 10(9)(8i) + 5(3)(16) \underbrace{-32i}_{} \ \tfrac{1}{2}$$

$$= 243 \underbrace{-810i}_{\frac{1}{2}} \underbrace{-1080}_{\frac{1}{2}} \underbrace{+720i}_{1} \underbrace{+240}_{1} -32i$$

$$= \underbrace{-597-122i}_{} \ 1$$

Find the 7^{th} term of $(8b-11g)^{25}$. **Your final coefficient may be in factored form as shown in lecture.**

SCORE: _____ / 5 PTS

$${}_{25}C_6(8b)^{25-6}(-11g)^6$$

$$= \frac{25!}{6!\,19!}\underbrace{(8b)^{19}}_{\frac{1}{2}}\underbrace{(-11g)^6}_{} \ \tfrac{1}{2}$$

PLUS $\frac{1}{2}$ IF YOUR FINAL ANSWER IS POSITIVE

$$= 1 \ \boxed{\frac{25\cdot \cancel{24}\cdot 23\cdot 22\cdot \cancel{21}^{7}\cdot \cancel{20}^{2}\cdot \cancel{19!}}{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1\cdot\cancel{19!}}}\ 8^{19}\,11^6\,b^{19}g^6$$

$$= \tfrac{1}{2}\ \underbrace{25\cdot 23\cdot 22\cdot 7\cdot 2}_{}\cdot 8^{19}\cdot 11^6 b^{19}g^6 = 177100\cdot \underbrace{8^{19}}_{\frac{1}{2}}\cdot\underbrace{11^6}_{\frac{1}{2}}\underbrace{b^{19}g^6}_{1}$$